Assignment 0

Advanced Computing Concepts for Engineering

Due January 31, 2020

Note that the work that you turn in for this assignment must represent your individual effort. You are welcome to help your fellow students to understand the material of the course and the meaning of the assignment questions, however, the answer that you submit must be created by you alone.

Please consider preparing your assignment with a typesetting program such as TeX, LaTeX, LyX, Scientific Word, or MS Word.

Q0 Signatures and behaviours

Suppose that we have a system with two inputs, each of type $\{0, 1, 2, 4\}$ and one output of type $\{\mathfrak{true}, \mathfrak{false}\}$.

(a) What could be a signature for the system.

(b) How many behaviours belong to this signature.

(c) How many specifications have this signature?

(d) Which of these behaviours are accepted by a specification that says that the output is true if and only if the first input is less than the second.

Q1 Writing specifications.

Suppose n is some fixed natural number greater than 0. Then $\mathbb{Z}^n = \left(\{0, ..n\} \stackrel{\text{tot}}{\to} \mathbb{Z}\right)$ is the set of all sequences of integers of length n. Let

$$\Sigma = \{\text{“}x\text{”} \mapsto \mathbb{Z}, \text{“}a\text{”} \mapsto \mathbb{Z}^n, \text{“}x'\text{”} \mapsto \mathbb{Z}, \text{“}a'\text{”} \mapsto \mathbb{Z}^n\}$$

Where x and x' represent, respectively, the initial and final values of a program variable x, and a and a' represent, respectively, the initial and final values of a program variable a.

Using angle-bracket notation, give a mathematical version of the following specifications:

(a) The final value of x is the largest of the items in the initial value of a.

(b) Every item of a (except the first) is larger than the previous item.

(c) If the initial value of x is in the set $\{0, ..n\}$, the final value of a is the same as its initial value except that item x is 42. Otherwise, if the initial value of x is not in the set $\{0, ..n\}$, it does not matter what the final values are.

(d) The initial value of x must be positive.

Q2 Refinement

(a) Make a table of all (16) behaviours belonging to $\Sigma \dagger \Sigma$ where[1]

$$\Sigma = \{\text{“}x\text{”} \mapsto \{1, 2, 3, 4\}\}$$

For each behaviour, indicate whether it is accepted (✓) or rejected ($\times$) by each of the following specifications (on $\Sigma \dagger \Sigma$)

$$\begin{array}{rcl}
a & = & \langle x > 3 \Rightarrow x' > 1 \rangle_{\Sigma \dagger \Sigma} \\
b & = & \langle x > 2 \Rightarrow x' > 1 \rangle_{\Sigma \dagger \Sigma} \\
c & = & \langle x > 3 \Rightarrow x' > 2 \rangle_{\Sigma \dagger \Sigma} \\
e & = & \langle x > 2 \Rightarrow x' > 2 \rangle_{\Sigma \dagger \Sigma} \\
f & = & \langle x > 1 \Rightarrow x' > 2 \rangle_{\Sigma \dagger \Sigma} \\
g & = & \langle x > 2 \Rightarrow x' > 3 \rangle_{\Sigma \dagger \Sigma} \\
h & = & \langle x > 1 \Rightarrow x' > 3 \rangle_{\Sigma \dagger \Sigma} \\
i & = & \langle x > 1 \land x' > 3 \rangle_{\Sigma \dagger \Sigma} \\
\textbf{magic} & = & \langle \mathfrak{false} \rangle_{\Sigma \dagger \Sigma} \\
\textbf{abort} & = & \langle \mathfrak{true} \rangle_{\Sigma \dagger \Sigma}
\end{array}$$

(b) What are the refinement relations between the specifications in part (a). Illustrate these relationships with a Hasse diagram.[2]

(c) Which of these specifications are deterministic? Which are implementable?

[1] $\Sigma \dagger \Sigma$ is $\{\text{“}x\text{”} \mapsto \{1, 2, 3, 4\}, \text{“}x'\text{”} \mapsto \{1, 2, 3, 4\}\}$

[2] You can look up Hasse diagrams on Wikipedia.