Assignment 1

Advanced Computing concepts for Engineering

Due Feb 14th, 2020 at 11:00am sharp.

Note that the work that you turn in for this assignment must represent your individual effort. You are welcome to help your fellow students to understand the material of the course and the meaning of the assignment questions, however, the answer that you submit must be created by you alone.

Submission may be on paper or via online.mun.ca. Handwritten submissions must be on paper.

Q0 [10] Given $\Sigma = \{ x^* \mapsto \mathbb{R}, y^* \mapsto \mathbb{R}, z^* \mapsto \mathbb{R} \}$, use the alternation law (and other laws) to implement

$$\langle (x < z \land y' = -1) \lor (x > z \land y' = 1) \lor (x = z \land y' = 0) \rangle$$

with a command. Show each step of the derivation.

Q1 [20] Binary search. Given a constant N > 0 and a constant function $C : \{0, ..., N\} \xrightarrow{\text{tot}} \mathbb{N}$, that is sorted (nondecreasing). We want to see whether there is an item of C that equals x. I'll use the notation $C\{p, ...r\}$ for the set of items with indecies in set $\{p, ...r\}$, i.e., $C\{p, ...r\} = \{k \in \{p, ...r\} \cdot C(k)\}$. Suppose we have variables p, q, r of type \mathbb{N} . Our specification is

$$f = \langle 0 \le p' < N \land (x \in C\{0, ..N\} \Leftrightarrow x = C(p')) \rangle$$

We can 'generalize' f as

$$g = \left\langle 0 \le p < r \le N \Rightarrow \left(\begin{array}{c} 0 \le p' < N \\ \wedge \quad (x \in C\{p, ..r\} \Leftrightarrow x = C(p')) \end{array} \right) \right\rangle$$

(a) [5] Find an initialization command i such that $f \sqsubseteq i; g$.

(b) [10] Implement g recursively using an alternation. Try to ensure that each iteration reduces r - p to roughly half its value. [Hint: Because C is nondecreasing: If $0 \le p < q < r \le N$ and C(q) > x, $x \in C\{p, ..r\} \Leftrightarrow x \in C\{p, ..r\}$ defined as $C(q) \le x$, $x \in C\{q, ..r\}$.] Be sure to justify each step of your derivation.

(c) [5] Apply (the incomplete version of) the while law to implement g with a while-command. However you should still informally check that your loop will terminate, so state a bound expression for the loop.

Q2 [20] Russian peasant multiplication

Suppose x, y, and z are natural number variables. We want to implement: $f = \langle z' = x \times y \rangle$ without using a multiplication.

(a) [5] Find a 'generalization' g that will work with the initialization command z := 0. I.e. we want $f \sqsubseteq z := 0; g$.

(b) [10] Implement g recursively using an alternation command. We would like the time to be proportional to the $\log_2 y$. You will find the following identities useful: $x \times y = x + x \times (y - 1)$ if y > 0. And $x \times y = 2 \times x \times y/2$ if y is even. Be sure to justify each step of your derivation.

(c) [5] Apply (the incomplete version of) the while law to implement g with a while-command. However you should still informally check that your loop will terminate. State a bound for your loop.